B-Math-II Back Paper Exam ; Analysis 3

Time : 3.00 hrs; Max Mark:45 ; 24 January 2022

1. Let $S \subset \mathbb{R}^n$ be open and connected. Let $\phi : S \to \mathbb{R}$ be continuously differentiable. Let $\underline{a}, \underline{b} \in S$ and α a piecewise smooth path joining $\underline{a}$ and $\underline{b}, \alpha : [0,1] \to S$. Show that $\int \nabla\phi \cdot d\alpha$ is independent of the path. (15)

2. Calculate the work done by a force field

$$f(x,y) := (y+3x)\vec{i} + (2y-x)\vec{j}$$

in moving a particle once around the ellipse $4x^2 + y^2 = 4$. (15)

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x,y) := e^{\frac{y-x}{y+x}}, (x,y) \in \mathbb{R}^2.$$

Evaluate $\iint\limits_S f(x,y)dxdy$ where S is the triangle

$$S := \{(x,y) : 0 \leq x+y \leq 2, x \geq 0, y \geq 0.\}$$

(15)

4. Let $f_n : [a,b] \to \mathbb{R}$ be continuous functions. Suppose f_n converge to f uniformly on $[a,b]$. Show that

$$\int_a^b f(x)dx = \lim_{n\to\infty} \int_a^b f_n(x)dx.$$

(15)

5. Le $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f = (f_1, f_2), f_1(x,y) := e^x \cos y, f_2(x,y) := e^x \sin y$.
a) Is f one-one on $\mathbb{R}^2$? Prove your answer.
b) Find the inverse of f in a neighbourhood of the point $(1,0)$.
c) Find the image of the coordinate axes under the map f.
(5+5+5)